

# Vertex-transitive graphs with the minimal diameter

## Intro

The proposed task is closely related to the famous degree/diameter problem [1, 2] for vertex-transitive graphs. A *vertex-transitive graph* is a graph such that every two nodes are equivalent under some element of its automorphism group [3]. As a consequence, such graphs are regular and are sometimes called node symmetric. All vertex-transitive graphs up to 47 nodes are known [4].

## Task

**Given:**  $n \in \mathbb{N}$  and  $d \in \mathbb{N}$ , where  $48 \leq n \leq 128$ ,  $d$  is even,  $4 \leq d \leq 16$ .

**Find** a connected  $d$ -regular vertex-transitive graph  $G$  with  $n$  nodes.

**Minimize** the diameter of  $G$ .

## Motivation

Such graphs are used in designing of data center networks. The degree  $d$  is the number of ports in nodes (radix). The diameter defines the latency of the data transmission. Vertex-transitivity provides uniformity of the network and helps to balance the load.

## Baseline

Edges of the *circulant graph*  $Ci_n(L)$  on nodes  $V = \{0, 1, \dots, n-1\}$  and an offset list  $L \subseteq \{1, 2, \dots, \lfloor n/2 \rfloor\}$  are defined as follows. Nodes  $i$  and  $j$  are adjacent iff  $j = (i+x) \bmod n$  or  $j = (i-x) \bmod n$  for some  $x \in L$ .

Circulant graphs are vertex-transitive and it is easy to solve the task for them [6, 7]. We provide a CSV-file with circulant graphs with the minimal diameter for all  $48 \leq n \leq 128$  and  $4 \leq d \leq 16$ .

## Scoring

You should provide an archive with vertex-transitive graphs that have a better diameter than circulant graphs. For each pair of  $n$  and  $d$ , your score

$$\text{Score}(n, d) = \max\{C(n, d) - X(n, d), 0\},$$

where  $C(n, d)$  is the minimal diameter for circulant graphs and  $X(n, d)$  is the diameter of your vertex-transitive graph. The total score:

$$\text{TotalScore} = \sum_{\substack{n \in \{48, 49, \dots, 128\} \\ d \in \{4, 6, \dots, 16\}}} \text{Score}(n, d).$$

The solution with the highest score wins.

Please, save every example of a graph in a file with name  $\langle n \rangle \langle d \rangle . \text{adj}$  (where  $\langle n \rangle$  is the nodes number and  $\langle d \rangle$  is the degree of the graph) as an adjacency list in the NetworkX format [8].

## Hints

Cayley graphs are vertex-transitive.

The vertex-transitivity may be checked with the following criteria. A connected graph  $G$  is vertex-transitive iff all its vertex-deleted subgraphs  $G \setminus v$  are isomorphic [9].

## References

- [1] M. Miller, J. Širáň. Moore graphs and beyond: A survey of the degree/diameter problem. *Electron. J. Combin. DS14* (2nd. edition. May 2013) 92 pp.
- [2] Combinatorics Wiki: [The Degree/Diameter Problem](#).
- [3] <https://mathworld.wolfram.com/Vertex-TransitiveGraph.html>
- [4] G. Royle, D. Holt. [Vertex-transitive Graphs On Fewer Than 48 Vertices](#).
- [5] <https://mathworld.wolfram.com/CirculantGraph.html>
- [6] Combinatorics Wiki: [The Degree Diameter Problem for Circulant Graphs](#).
- [7] Graphs: [Circulant graphs](#)
- [8] NetworkX: [Adjacency List](#).
- [9] Carsten Thomassen. A characterization of locally finite vertex-transitive graphs. *Journal of Combinatorial Theory, Series B, Volume 43*.