Antenna Selection

Business background and requirements

In *multiple-input-single-output (MISO)* antenna arrays, energy efficiency is a critical concern. A radical approach to energy conservation is switching off some antennas. While this is typically done in a fixed, cell-specific, manner, selecting the active antenna set in a user-specific way leads to an interesting optimization problem.

Most energy is consumed not by transmission itself but by operational overhead costs that persist even during low-traffic periods (e.g., at night). This means the amount of energy saved can be considered equal for all antennas we turn off, so we just decide that we want, e.g., 50% antennas turned off. There is a trade-off between operating power, fewer active antennas reduce the energy consumption, and *Beamforming (BF) Gain,* more antennas increase the BF gain, defined as the modulus of scalar product between the channel vector and antenna weights vector.

Our goal is to select either 25% or 50% of N antennas to turn off while minimizing performance loss. In the trivial case of single-stream transmission ($L = 1$), one can simply turn off the "weakest" antennas, those with minimal absolute value of the vector elements. The problem becomes more challenging when dealing with a weight matrix ($L > 1$) under per-antenna power constraint, where the squared norm of each row (corresponding to an antenna) is limited.

Informal problem statement

Strike out some rows in a complex-valued matrix $V \in \mathbb{C}^{N \times L}$, so that the resulting matrix is as close to an orthonormal basis as possible.

Formal problem statement

1. We have a complex-valued matrix $V \in \mathbb{C}^{N \times L}$ with orthonormal columns: $V^* V = \frac{1}{L} I_{L \times L}$.
2. For each row, we introduce binary variable: $x_n = 1$ if the antenna n is active, 0 otherwise, $n = 1, \dots, N$.
3. Define new matrix $W \in \mathbb{C}^{N \times L}$ by the rule: $w_{nj} = z\, v_{nj} \cdot x_n$ and calculate the positive scalar $z \in \mathbb{R}_+$ in such a way that $\max_n z \sum_{j=1}^{L} \left| w_{nj} \right|^2 = P$ where constant P is a given parameter.
4. Calculate the power on antennas: $s_n = \sum_{j=1}^{L} \left| v_{nj} \right|^2$ and $p_n = \sum_{j=1}^{L} \left| w_{nj} \right|^2$.
5. Effective channel after selection: $V_{eq}(x) = V^* W$, size $L \times L$.
6. σ: noise power, may be assumed 0.1.

Cardinality Constraint

At least 25 or 50% of antennas must be turned off: $\sum_{n=1}^{N} x_n \le K$, $K = 3N/4$ or $K = N/2$.

Alternative Objectives

1. **BF gain maximization.** Maximize $U_{BF} = \mathrm{Tr}\left(V_{eq} V_{eq}^*\right)$.
2. **Interference minimization.** Minimize sum of non-diagonal terms $U_I = \left\| V_{eq} \right\|_F^2 - \mathrm{Tr}\left(V_{eq} V_{eq}^*\right)$.
3. **General objective.** Maximize $U_G = \det\left(V_{eq} V_{eq}^* + \sigma I\right)$.

Objectives 1 and 2 represent extreme cases of the General objective. When $\sigma > 1$, it means that there is a lot of noise in the system, so adding some interference (non-diagonal terms in $V_{eq} V_{eq}^*$) is not so important and BF gain (diagonal i of $V_{eq}V_{eq}^*$) is the main aspect for optimization. When $\sigma \ll 1$, interference determines the performance, so the main goal is to preserve orthogonality. This leads to simplified solutions for objectives 1 and 2:

Heuristic solutions

H1. Delete weakest antennas: Sort antennas by p_n (ascending) and select the first K indices to deactivate.

H2. Iterative interference-based deletion: Deactivate antennas one-by-one, we delete antennas based on their contribution to interference. For $L = 2$, we select antenna n that solves $\min_n \left|\sum_{k=1}^{N} v_{k1}^* v_{k2} - v_{n1}^* v_{n2}\right|$.

Input data

$N \geq 1000,\ P = 1,\ L = 2, 3, \ldots 10,$

Set $\sigma = 0.1$ for solving Task 3.

The data for original matrix V will be made available.

Scoring

Your solution W for any of the Objectives will be scored as:

$$S(W) = \max\big(U(H1), U(H2)\big) - U(W).$$

We expect you to achieve at least 5% improvement in any of the Objectives compared to the best of Heuristic Solutions 1 and 2.

Preferences

We would appreciate theoretical estimates for the performance of the proposed solutions, at least in the high interference approximation, task 2. For theoretical considerations one or both of the further simplifications can be applied: $L = 2;\ v_{ij} = \frac{1}{\sqrt{N}} e^{i\phi_{ij}}$.

Reference

An Adaptive Approach for the Joint Antenna Selection and Beamforming Optimization

https://ieeexplore.ieee.org/document/8766818